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BOUNDARY-LAYER INSTABILITY AT SUBSONIC SPEEDS

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ABSTRACT

Results of experimental investigations of instability leading to transition in the subsonic boundary-layer flow along a concave plate and the subsonic boundary-layer flow along a flat plate with an isolated roughness element are presented. The feature common to both flows is the generation of streamwise vortices, which in turn make the boundary layer three-dimensional. Results of hot-wire measurements and smoke observations are used to obtain a collective view on the development of two-dimensional disturbances under the influence of boundary-layer three-dimensionality.

INTRODUCTION

It is now known through the investigations of Schubauer and his co-workers that the transition in the boundary layer along a smooth flat plate is preceded by the appearance of two-dimensional disturbances of the type predicted by the linearized theory of laminar instability. In the course of downstream development, however, the disturbances exhibit variations in amplitude along a spanwise direction, which is parallel to the leading edge of the plate, before eventually laminar flow breaks down. This development of disturbances into three-dimensional configuration is traced back to the small preexisting three-dimensionality such as spanwise variations of mean flow, but it is nevertheless regarded as an inherent phenomena constituting an important aspect of boundary layer instability.

With a view to minimizing the undesirable effect of residual three-dimensionality, Klebanoff, Tidstrom and Sargent[8] carried out an experiment in which disturbances with controlled spanwise variation in amplitude were introduced in a boundary layer along a flat plate having barely detectable spanwise variation in mean velocity. The disturbances were created by Schubauer's technique of vibrating ribbon, underneath which strips of cellophane tape were placed at regular intervals on the plate. As the disturbances grew downstream, the spanwise variation in amplitude was greatly accentuated, until eventually the breakdown of laminar flow appeared in the section of the peak of amplitude variation. Tani and Komoda,[9] on the other hand, made an observation of downstream development of controlled two-dimensional disturbances in a boundary layer having a controlled spanwise variation in mean velocity, which was produced by means of small wings placed at regular intervals outside the boundary layer. When the mean-velocity variation was small, the development of disturbances was similar to that observed by Klebanoff, Tidstrom and Sargent, and the breakdown appeared first in the section of the peak of mean-velocity variation. When the mean-velocity variation was large, however, the disturbances grew considerably in the section close to that of the valley of mean-velocity variation, resulting in the first appearance of breakdown there.

The paper presents results of the experimental investigations concerning the instability leading to transition in the boundary layer along a concave plate and the boundary layer along a flat plate with an isolated roughness element. To make the situation as simple as possible, the effects of pressure gradient and compressibility are eliminated. In common with the experimental arrangement employed by Tani and Komoda, the two boundary-layer flows under consideration are characterized by generation of streamwise vortices, which in turn produce a spanwise variation in mean velocity. It is therefore hoped that the results of investigations will not only afford a clear insight into the mechanism of transition for the respective types of flow, but also suggest a collective view concerning the three-dimensional development of disturbances leading to transition.

As in the previous paper[7] presented at the Second Congress at Zürich, the investigations to be described were also conducted as the joint work of an informal group for boundary layer research, with its center in the Aeronautical Research Institute, University of Tokyo. The experiments were carried out with the cooperation of Mr. M. Iuchi and Mr. Y. Aihara on the boundary layer along a concave plate, and Dr. H. Komoda and Mr. Y. Komatsu on the boundary layer along a flat plate with a roughness element. Detailed account of the investigations will be published elsewhere by the individual members.

TRANSITION IN BOUNDARY LAYER ALONG CONCAVE PLATE

Measurements were made on the boundary layer along a concave plate with a radius of curvature of 300 cm mounted in a wind tunnel of 60 × 60 cm working section. By adjusting the wall opposite to the test wall, it was possible to eliminate the streamwise gradient of free-stream velocity (velocity outside the boundary layer) within the accuracy of measurements. The reference axes were taken in such a way that x was measured along the plate from the leading edge in the streamwise direction, y in the direction perpendicular to the plate, and z in the spanwise direction perpendicular to x and y. Most measurements were made at a free-stream velocity U_0 of 7 m per sec, where the free-stream turbulence level was of the order of 0.05 percent. Controlled two-dimensional disturbances, or Tollmien-Schlichting waves, were produced at a location $x = 52$ cm by passing an alternating current of a frequency of 60 cps through a thin phosphor-bronze ribbon in the presence of a steady magnetic field.

The boundary layer on a concave wall is apparently dynamically unstable with respect to certain types of vortices with axes in the streamwise direction. This instability was first predicted by Görtler's theoretical analysis[1] and then indirectly confirmed by Liepmann's measurements[2,3] on a concave wall, in which the critical Reynolds number at transition was considerably reduced as compared with the value on a flat plate. However, no observation has been made until quite recently to confirm directly the existence of vortices of the type predicted by theory. [7,10]

Figure 1 shows a typical result for the station $x = 50$ cm of the streamwise component of mean velocity U obtained by traversing a hot-wire probe in the z-direction at a fixed height y from the concave wall. The periodic variation of velocity is regarded as produced by the system of alternating streamwise vortices superposed on the Blasius flow along the wall. The wavelength of spanwise variation is about 1.8 cm, and the number of waves α contained within a distance of 2π cm is $\alpha = 2\pi/1.8 = 3.5$. Measurements made at different values of x and U_0 indicate that the wave number is independent of x, and nearly independent of U_0.

The results on wave number are presented in Fig. 2 by plotting the Görtler parameter $G = R_\theta(\theta/r)^{1/2}$ against the nondimensional wave number $\alpha\theta$, where θ is the momentum thickness of the boundary layer in the section C (see Fig. 1),

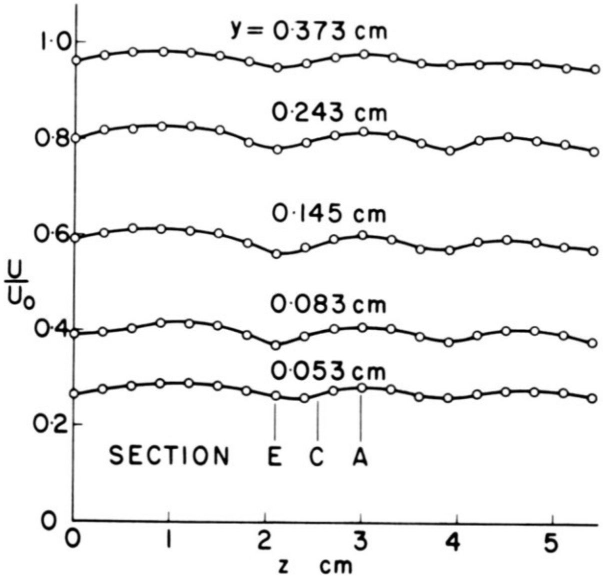


Fig. 1. Velocity distribution in spanwise direction at a fixed height above concave plate. $x = 50$ cm, $r = 300$ cm, $U_0 = 7$ m per sec. Sections A and E for peak and valley, respectively, of velocity distribution. Section C in the middle.

and $R_\theta = U_0\theta/\nu$ is the Reynolds number based on free-stream velocity and momentum thickness. Results of additional measurements on the concave plate $r = 100$ cm and previous measurements[10] on the concave plates $r = 500$ cm and 1000 cm are also entered. Comparison is made with the prediction by small-disturbance theory, which is originally due to Görtler,[1] and subsequently modified by Hämmerlin[5] and Smith.[6] For small values of $\alpha\theta$, Hämmerlin's result is more accurate than that of Görtler. In both theories the basic flow and disturbance are invariant in the x-direction, the disturbance being assumed in the form $F(y)\, e^{Bt} \cos \alpha z$, where t is the time. The disturbance is amplified or damped in the course of time according as B is positive or negative; it is neutral for $B = 0$. In Smith's theory, the basic flow is invariant in the x-direction, but the disturbance is assumed in the form $F(y)e^{bx} \cos \alpha z$. The disturbance is amplified or damped in the course of downstream development according as b is positive or negative; it is neutral for $b = 0$. The assumptions made in Smith's theory appear to be closer to the conditions of the present experiment.

It is seen from Fig. 2 that all the experimental points are located in the region where the disturbance is to be amplified according to theory. This affords an evidence to justify the observed result. Further evidences are obtained by

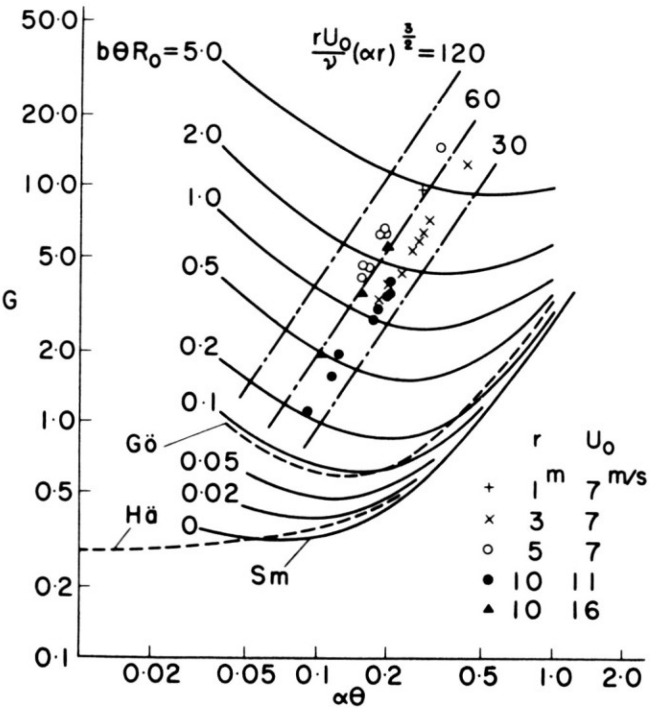


Fig. 2. Comparison of wave number of vortices observed on the concave wall with theoretical prediction. *Gö*, *Hä*, and *Sm* are the neutral curves due to Görtler, Hämmerlin, and Smith, respectively.

comparing with theory the distribution of disturbance amplitude in the y-direction as well as the rate of amplification b.

Thus, the observed spanwise variation of mean velocity is identified with that produced by the system of vortices predicted theoretically. However, the theory predicts nothing about the wave number that will actually come out for a given radius of curvature r. The observed values of α are 3.5, 3.5, 1.8 and 3.0 for r = 100, 300, 500 and 1,000 cm, respectively, almost independent of the free-stream velocity. Additional measurements were made either by adding a leading-edge portion of plane wall ahead of the test wall or by reducing the spanwise dimension of the wind tunnel by inserting false walls. But no appreciable change has been found in wave number. Since the concave plates r = 100 and 500 cm were tested in wind tunnels respectively different from that used for other plates (r = 300 and 1,000 cm), it may be conjectured that the wave number observed is the one inherent to the experimental arrangement provided that it is to be amplified according to theory.

Figure 3 shows the distribution across the boundary layer of mean velocity at various distances from the leading edge in sections A, E and C. Sections A and E correspond to a peak and a valley, respectively, of the spanwise variation

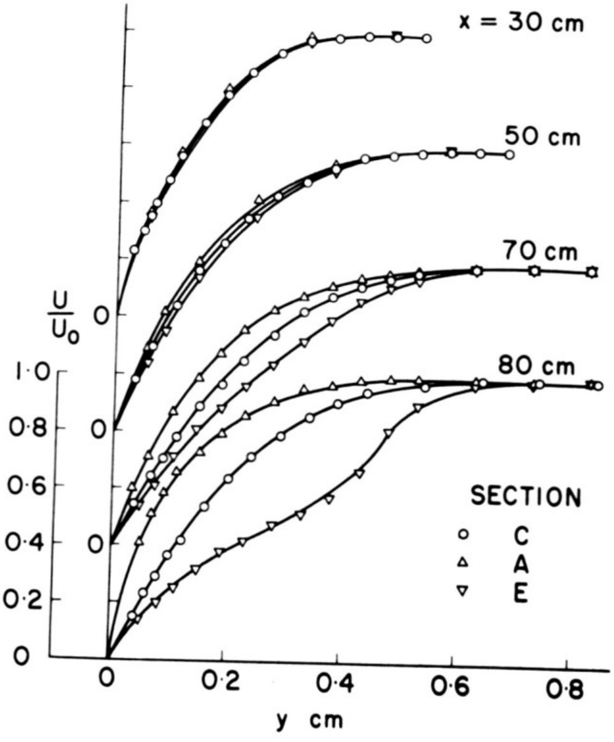


Fig. 3. Distribution across boundary layer along concave plate of mean velocity in sections A, E, and C at various distances from leading edge. r = 300 cm, U_0 = 7 m per sec, current to vibrating ribbon, i_r = 1.2 amp.

in mean velocity, and section C is taken in the middle of A and E (Fig. 1). In the course of downstream development a distortion proceeds in the velocity profile, which is very close to the Blasius profile at the upstream station $x = 30$ cm, until finally an unstable profile with inflection point is developed at downstream stations. The distortion of velocity profile for middle section C is indicative of the nonlinear growth of the streamwise vortices, while the development of a distinctly unstable profile for section E suggests the ensuing breakdown of laminar flow.

The Reynolds number, based on the free-stream velocity and boundary-layer displacement thickness for section C, was 700 at the location of the vibrating ribbon, so that the exciting frequency of 60 cps was the one to produce an amplifying Tollmien-Schlichting wave according to the linearized two-dimensional stability theory. Hot-wire measurements were therefore made of the streamwise velocity fluctuations to trace the downstream development of the excited wave. Figure 4 shows the distribution across the boundary layer of wave phase ϕ_f and rms wave intensity u_f at a frequency $f = 60$ cps at two stations ($r = 300$ and 1,000 cm), it may be conjectured that the wave number observed of the vibrating ribbon. The abscissa is made nondimensional by dividing the distance y from the plate by the boundary-layer thickness δ defined as equal to three times the displacement thickness. The results are for the case when the current to the ribbon slightly exceeds the linear limit below which u_f is in proportion to the initial disturbance. It is seen that the intensity distribution in valley section E is essentially of the type predicted by the Tollmien-Schlichting theory, accompanying a distinct phase shift at the height of intensity minimum, but that the distribution in peak section A is somewhat distorted. Especially significant is the rapid amplification of intensity in section E between the two stations, $x - x_r = 18$ and 28 cm. This wave amplification, together with the distortion of mean velocity profile shown in Fig. 3, indicates that the breakdown of laminar flow is approached. As a matter of fact, a characteristic distortion is observed in the oscillograph wave form of velocity fluctuation before the station $x - x_r = 50$ cm is reached.

The difference of wave intensity distribution in different longitudinal sections has been noticed in Tani and Komoda's measurements[9] on a boundary layer, in which streamwise vortices were introduced by a series of wings. Rapid amplification of wave intensity leading to breakdown was observed in the section corresponding to the peak of spanwise variation in mean velocity provided that the variation is not excessive. This correspondence is directly contrary to that described above, so that a question was raised whether the excited wave was initially stronger in section E than in section A. Additional measurements were therefore made by placing strips of tape, 0.9 cm long and 0.9 cm apart, on the plate beneath the vibrating ribbon, in such a way that the wave intensity was initially weaker in section E than in section A. The wave intensity observed at downstream stations were somewhat reduced, but no appreciable change was found in disparity between the two sections A and E. It may be speculated that the disparity in wave amplification is dependent on the three-dimensional structure of mean flow, possibly including the behavior of velocity components in the y- and z-directions.

Visual observations were also made on a concave plate of $r = 100$ cm mounted in another wind tunnel of 50 × 50 cm working section. Flow patterns were revealed by the smoke of paraffin either emitted from a slit on the plate or produced by electrically heating a platinum ribbon with liquid paraffin rubbed on, stretched at a small height y_s above the plate. Figures *5a* and *5b* show the photographs taken stroboscopically from above the plate. It is found that the distance between the adjacent streaks of smoke was in agreement with the wavelength of spanwise variation in mean velocity, and that the streaks are located in section E or section A according as smoke is introduced close to or at some height above the plate. Figure *5c* shows the superposition of the streaks observed in Figs. *5a* and *5b*.

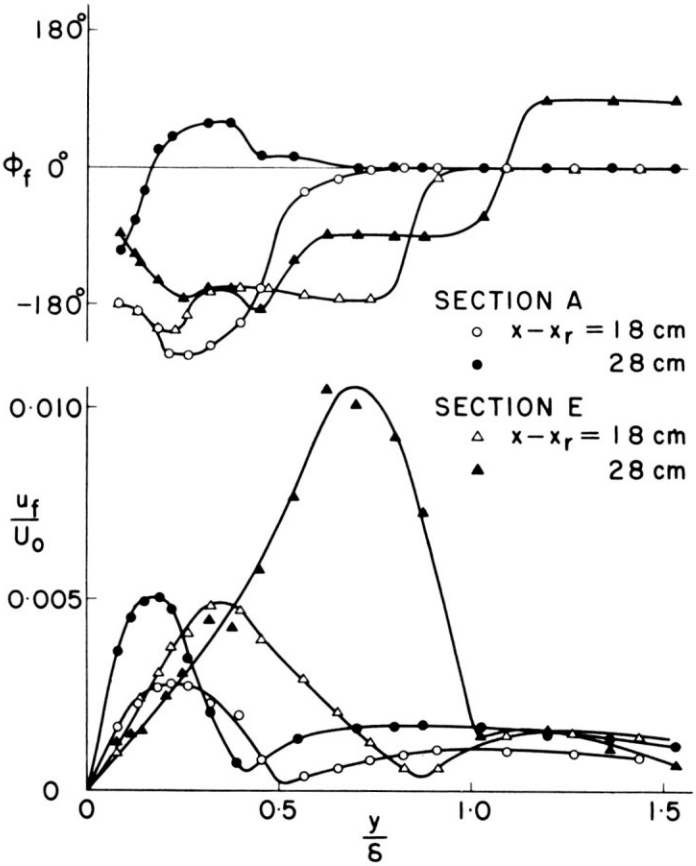
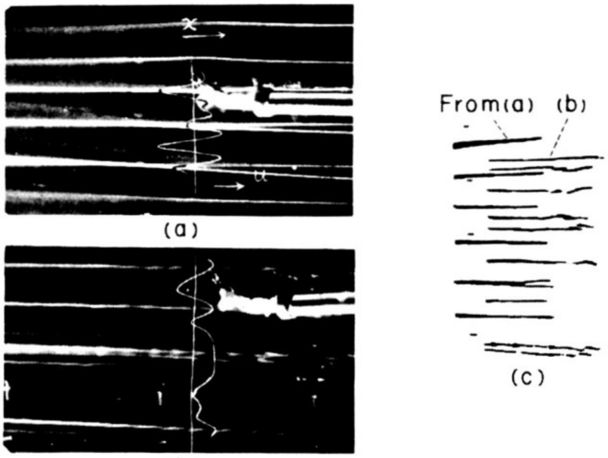


Fig. 4. Distribution across boundary layer along concave plate of wave phase and rms wave intensity at a frequency of 60 cps at two stations in sections A and E. $r = 300$ cm, $U_0 = 7$ m per sec. Location of vibrating ribbon, $x_r = 52$ cm, current to vibrating ribbon, $i_r = 1.2$ amp.



(a)

(b)

(c)

Fig. 5. Smoke streaks on the concave plate $r = 100$ cm, with a leading-edge portion of plane wall 25 cm in length. $U_0 = 5$ m per sec. (*a*) Photograph, $y_s = 0$, (*b*) photograph, $y_s = 0.3$ cm, (*c*) superposition of (*a*) and (*b*). Spanwise variation of mean velocity is entered with white lines on the photographs.

Figure 6 shows the photographs of smoke emitted from a location a little downstream of the vibrating ribbon, which produces amplifying waves. Oscillographic records of velocity fluctuation are also inserted, which were simultaneously obtained by means of a hot-wire probe placed at the corresponding height of the smoke. It is seen that the wave intensity is stronger in section E than in section A throughout the boundary-layer thickness.

Figure 7 shows the photograph taken similarly on a concave plate of $r = 300$ cm, the smoke being emitted close to the plate at a location 10 cm behind the vibrating ribbon.

The features of flow pattern revealed by smoke observation are in good agreement with those obtained by hot-wire measurements.

TRANSITION IN BOUNDARY LAYER ALONG FLAT PLATE WITH ROUGHNESS ELEMENT

Measurements were made on the boundary layer along a flat plate mounted in a wind tunnel of 20 × 60 cm working section having an adjustable wall for the elimination of streamwise pressure gradient. The reference axes were taken in such a way that x was measured along the plate from the leading edge in the streamwise direction, y in the direction perpendicular to the plate, and z in the spanwise direction perpendicular to x and y. Measurements were made at a free-stream velocity U_0 of 6.6 m per sec, where the free-stream turbulence level was of the order of 0.05 percent.

An isolated roughness element was represented by a single cylinder of 0.2-cm diameter and 0.2-cm height placed with its axis in y direction centrally on the flat plate at a location $x_k = 40$ cm, $z_k = 0$. Controlled two-dimensional waves were produced at a downstream location, $x_r = 75.5$ cm, by passing a current of a frequency of 45 cps through a thin phosphor-bronze ribbon in the presence of a magnetic field.

Gregory and Walker[3] appear to have been the first to make systematic investigations of the effect of an isolated roughness element. They observed

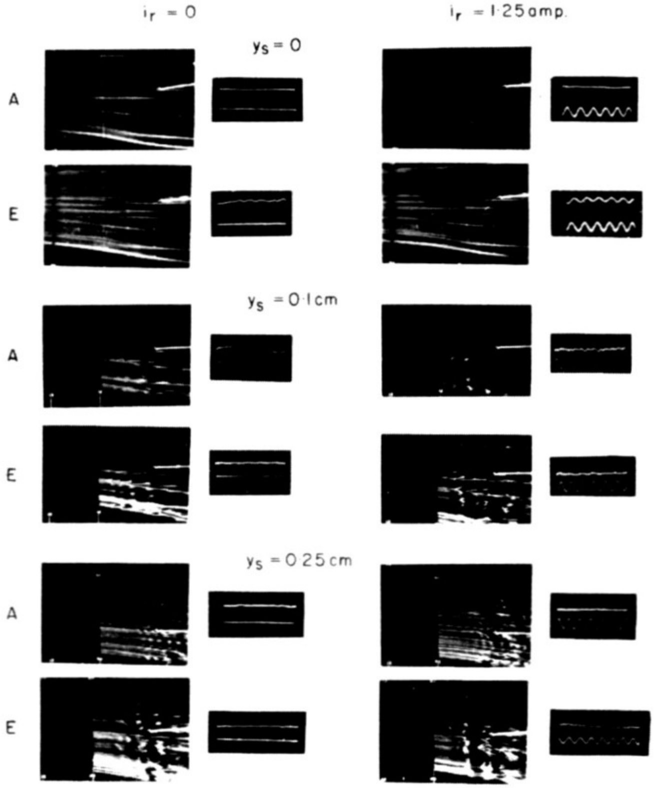


Fig. 6. Photographs of smoke streaks on the concave plate $r = 100$ cm, with a leading-edge portion of plane wall 25 cm in length. Vibrating ribbon at $x = 35$ cm (10 cm from juncture), smoke source at $x = 45$ cm, $U_0 = 5$ m per sec. Inserted oscillograms are obtained by a hot-wire probe placed at smoke height in section A or E. The lower trace indicates the current.

Fig. 7. Photograph of smoke streaks showing the wave development in section E. Concave plate $r = 300$ cm. $U_0 = 7$ m per sec. Vibrating ribbon (R) at $x = 52$ cm, smoke source (S) at $x = 62$ cm.

under certain conditions the china-clay records of twin streaks suggestive of vortex filaments in the boundary layer downstream of the roughness element, ultimately disintegrating into a wedge of turbulence. Closer examination by smoke revealed a horseshoe-shaped vortex wrapped round the front of the roughness and trailing downstream. The slightest increase in stream velocity or roughness height changed the pattern from twin streaks to a fully turbulent wedge originating at the roughness element. The free-stream velocity and the location and height of roughness element were selected for the present measurements in such a way that the intermediate situation was realized, in which the turbulent wedge began at a considerable distance from the roughness element but could be moved forward by passing a current through the vibrating ribbon.

Figure 8 shows the distribution of streamwise component of mean velocity U obtained by traversing a hot-wire probe in z-direction at a fixed height y above the wall at various distances $x - x_k$ from the roughness element. The spanwise variation of velocity is considered as produced mainly by a horseshoe-shaped vortex mentioned above superposed on the Blasius flow along the wall. It is to

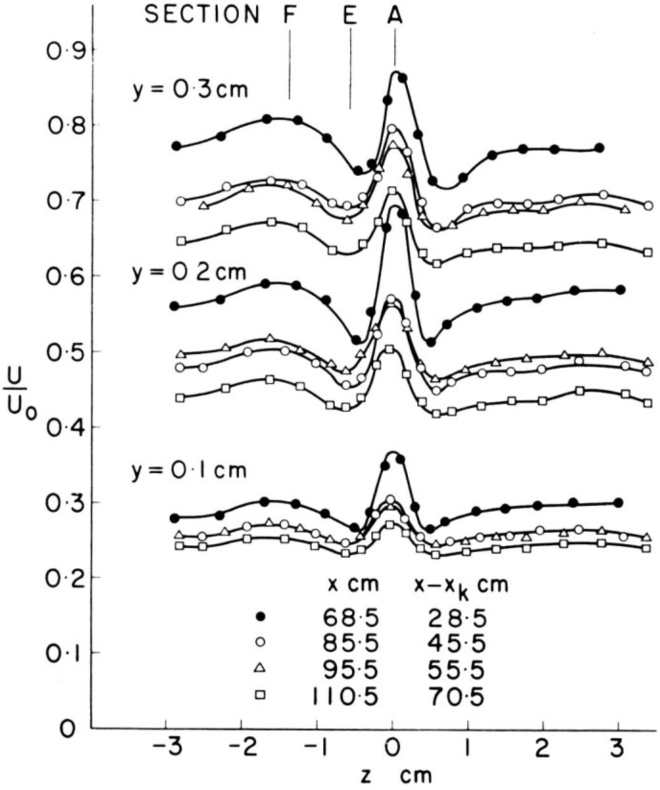


Fig. 8. Velocity distribution in spanwise direction at a fixed height above flat plate with roughness at $x_k = 40$ cm, $z_k = 0$. $U_0 = 6.6$ m per sec. Sections A, E, and F for central peak, valley and outside peak, respectively, of velocity distribution. No current to vibrating ribbon.

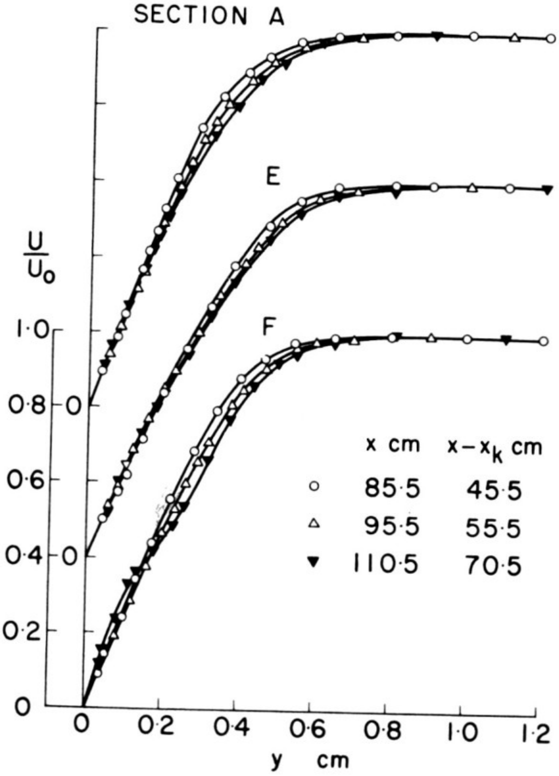


Fig. 9. Distribution across boundary layer along flat plate with roughness of mean velocity in sections *A*, *E*, and *F* at various distances from roughness. $x_k = 40$ cm, $U_0 = 6.6$ m per sec, current to vibrating ribbon, $i_r = 3$ amp.

be noticed that the distance between valleys, and also that between outside peaks, of velocity variation remain almost unchanged in downstream development. For later reference, the sections for the central peak, the valley and the outside peak are denoted by *A*, *E* and *F*, respectively.

Figures 9 and 10 show the distribution across the boundary layer of mean velocity U, and rms wave intensity u_f at a frequency $f = 45$ cps, respectively, in sections *A*, *E* and *F* at various distances from the roughness element. The results are for the case when the current to the ribbon exceeds the linear limit below which u_f is in proportion to the initial disturbance. The distribution of mean velocity is very close to the Blasius profile at the upstream station, but undergoes some distortion in the course of downstream development, the distortion being most remarkable in section *F*. It is to be noticed, however, that no appreciable distortion takes place so long as the current remains within the linear limit.

The Reynolds number, based on the free-stream velocity and boundary-layer displacement thickness, was 700, 920, and 850 in sections *A*, *E* and *F*, respectively, at the location of the vibrating ribbon, and the exciting frequency of

45 cps was the one to produce an amplifying Tollmien-Schlichting wave according to the linearized two-dimensional stability theory. The distributions of wave intensity in peak sections *A* and *F* are essentially of the type predicted by theory, but the distribution in valley section *E* is somewhat different and characterized by an *M*-shaped curve. This disparity in intensity distribution is in general agreement with that observed by Tani and Komoda[9] on the boundary layer in the presence of streamwise vortices introduced by wings placed at regular intervals outside the boundary layer. In section *A*, the wave is neither amplified nor damped, until finally a change-over in distribution takes place to the *M*-shaped curve. In section *E*, the *M*-shaped distribution is maintained in the course of wave amplification, the peak closer to the wall becoming higher.

In section *F*, however, the wave is strikingly amplified, beyond comparison with other sections. The amplification is so rapid that a distortion takes place in oscillograph wave form of velocity fluctuation in a layer close to the wall

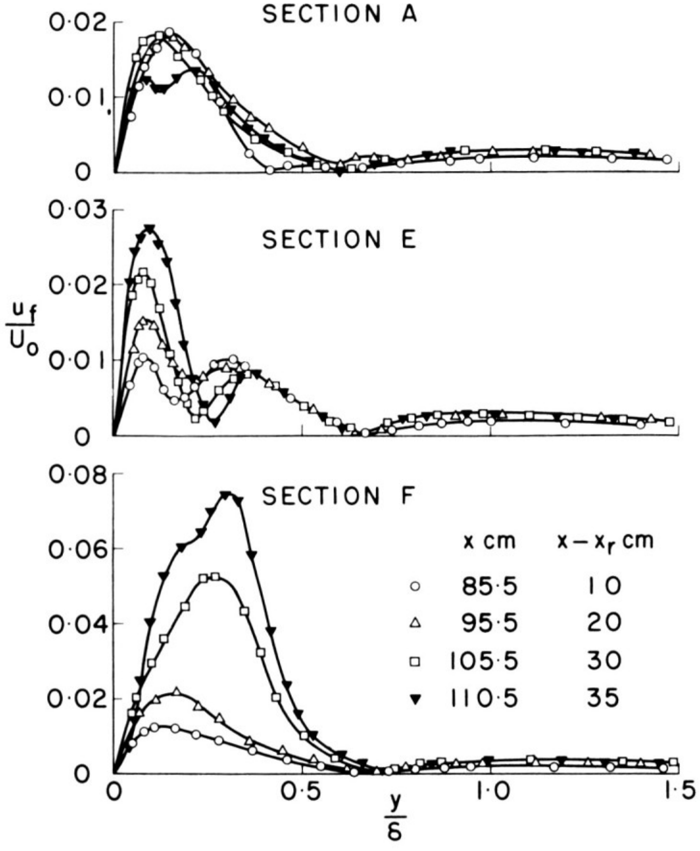


Fig. 10. Distribution across boundary layer along flat plate with roughness of rms wave intensity at a frequency of 45 cps in sections *A*, *E*, and *F* at various distances from vibrating ribbon. U_0 = 6.6 m per sec, x_r = 75.5 cm, current to vibrating ribbon, i_r = 3 amp. δ is three times displacement thickness.

beginning from the station $x - x_r = 30$ cm. This evolution corresponds to the distortion in mean velocity profile close to the wall as already mentioned in connection with Fig. 9. At further downstream station $x - x_r = 45$ cm, the oscillograph wave form is interspersed with spikes, characteristic of breakdown of laminar flow. This appears to explain why turbulence takes place in the form of a wedge, which originates from just outside of the horseshoe vortex.

CONCLUSION

The feature common to the two flows under investigation, the boundary layer along a concave plate and the boundary layer along a flat plate with an isolated roughness element, is the generation of streamwise vortices, which in turn produce a spanwise variation in mean velocity. Transition from laminar to turbulent in the respective boundary-layer flows may be interpreted as the evolution of disturbances leading to breakdown under the influence of boundary-layer three-dimensionality.

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